SEQUENCES AND SERIES

1   For each of the following arithmetic series, write down the common difference and find the value of the 40th term.

   a $4 + 10 + 16 + 22 + \ldots$ **b** $30 + 27 + 24 + 21 + \ldots$ **c** $8.9 + 11.2 + 13.5 + 15.8 + \ldots$

2   For each of the following arithmetic series, find an expression for the nth term in the form $a + bn$.

   a $7 + 9 + 11 + 13 + \ldots$ **b** $\frac{1}{6} + 1\frac{1}{2} + 2\frac{5}{6} + 4\frac{1}{6} + \ldots$ **c** $17 + 9 + 1 + (-7) + \ldots$

3   Find the sum of the first 30 terms of each of the following arithmetic series.

   a $8 + 12 + 16 + 20 + \ldots$ **b** $60 + 53 + 46 + 39 + \ldots$ **c** $7\frac{1}{4} + 8\frac{3}{4} + 10\frac{1}{4} + 11\frac{3}{4} + \ldots$

4   Given the first term, a, the last term, l, and the number of terms, n, find the sum of each of the following arithmetic series.

   a $a = 60, l = 136, n = 20$ **b** $a = 100, l = 84.5, n = 32$ **c** $a = 28, l = -20, n = 17$

5   Given the first term, a, the common difference, d, and the number of terms, n, find the sum of each of the following arithmetic series.

   a $a = 2, d = 9, n = 48$ **b** $a = 100, d = -5, n = 36$ **c** $a = 19, d = 13, n = 55$

6   Given the first term, a, the common difference, d, and the last term, l, find the sum of each of the following arithmetic series.

   a $a = 8, d = 3, l = 65$ **b** $a = 3.4, d = 1.2, l = 23.8$ **c** $a = 22, d = -8, l = -226$

7   The first and third terms of an arithmetic series are 21 and 27 respectively.

   a   Find the common difference of the series.

   b   Find the sum of the first 40 terms of the series.

8   The nth term of an arithmetic series is given by $7n + 16$.

   Find the first term of the series and the sum of the first 35 terms of the series.

9   The second and fifth terms of an arithmetic series are 13 and 46 respectively.

   a   Write down two equations relating the first term, a, and the common difference, d, of the series.

   b   Find the values of a and d.

   c   Find the 40th term of the series.

10   The third and eighth terms of an arithmetic series are 72 and 37 respectively.

   a   Find the first term and common difference of the series.

   b   Find the sum of the first 25 terms of the series.

11   The fifth term of an arithmetic series is 23 and the sum of the first 10 terms of the series is 240.

   a   Find the first term and common difference of the series.

   b   Find the sum of the first 60 terms of the series.

12   **a**   Prove that the sum of the first n natural numbers is given by $\frac{1}{2}n(n + 1)$.

   b   Find the sum of the natural numbers from 30 to 100 inclusive.

13 Write down all the terms in each of the following series summations.

  a $\displaystyle\sum_{r=1}^{5} (2r+3)$ **b** $\displaystyle\sum_{r=1}^{9} (18-3r)$ **c** $\displaystyle\sum_{r=4}^{10} (4r-1)$ **d** $\displaystyle\sum_{r=11}^{18} (10-\tfrac{1}{2}r)$

14 Evaluate

  a $\displaystyle\sum_{r=1}^{20} (3r+1)$ **b** $\displaystyle\sum_{r=1}^{45} (90-2r)$ **c** $\displaystyle\sum_{r=3}^{30} (4r+7)$ **d** $\displaystyle\sum_{r=10}^{50} \left(\tfrac{r+2}{4}\right)$

15 Given that $\displaystyle\sum_{r=1}^{n} (4r-6) = 720,$ find the value of n.

16 Find the sum of

  a all even numbers between 2 and 160 inclusive,

  b all positive integers less than 200 that are divisible by 3,

  c all integers divisible by 6 between 30 and 300 inclusive.

17 An arithmetic series has common difference −11 and tenth term 101.

  a Find the first term of the series.

  b Find the sum of the first 30 terms of the series.

18 The first and fifth terms of an arithmetic series are 17 and 27 respectively.

  a Find the common difference of the series.

  Given that the rth term of the series is 132,

  b find the value of r,

  c find the sum of the first r terms of the series.

19 The sum of the first six terms of an arithmetic series is 213 and the sum of the first ten terms of the series is 295.

  a Find the first term and common difference of the series.

  b Find the number of positive terms in the series.

  c Hence find the maximum value of S_n, the sum of the first n terms of the series.

20 The sum, S_n, of the first n terms of an arithmetic series is given by $S_n = 2n^2 + 5n$.

  a Evaluate S_8.

  b Find the eighth term of the series.

  c Find an expression for the nth term of the series.

21 The first three terms of an arithmetic series are $(k + 2)$, $(2k + 3)$ and $(4k − 2)$ respectively.

  a Find the value of the constant k.

  b Find the sum of the first 25 terms of the series.

22 The fifth, sixth and seventh terms of an arithmetic series are $(5 − t)$, $2t$ and $(6t − 3)$ respectively.

  a Find the value of the constant t.

  b Find the sum of the first 18 terms of the series.